1 Mixed-mode fracture

In this exercise, we will study the kink angle observed during mixed-mode fracture. Let us consider a through crack in a plate submitted to a pure mode-II loading τ_{xy}. As illustrated in Figure 1, the crack will rotate with a kink angle α due to mode-mixity.

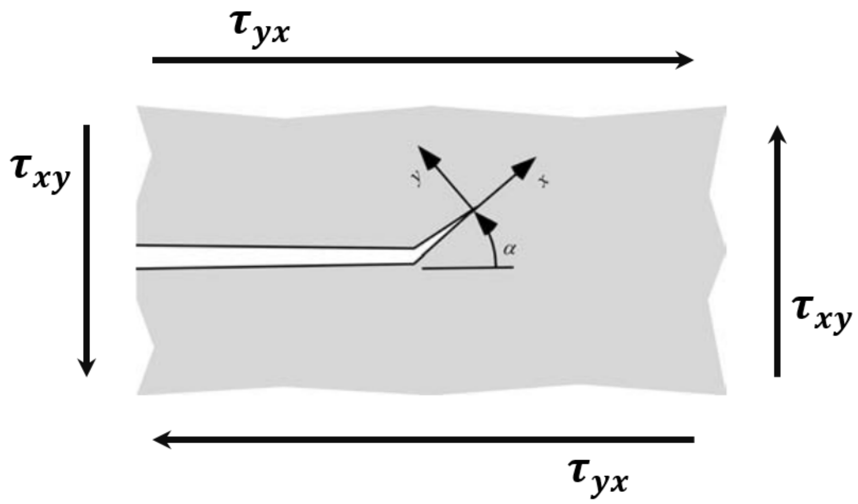

Figure 1: Mode-II crack kinking with an angle α. loading conditions.

The stress intensity factors K_I^*,K_{II}^*at the tip of the kink can be written as function of K_{II} before the kink

$$K_I^* = -\frac{3}{4}\left[\sin(\frac{\alpha}{2}) + \sin(\frac{3\alpha}{2})\right] K_{II} \tag{1}$$

$$K_{II}^* = \frac{1}{4}\left[\cos(\frac{\alpha}{2}) + 3\cos(\frac{3\alpha}{2})\right] K_{II} \tag{2}$$

under the assumption of an infinitesimal kink.

Question 1

List and present the three different criteria describing crack kinking in case of mixed-mode fracture.

Question 2

Compute the kink angle according to the principle of local symmetry (PLS). Hint:

$$\cos(3x) = \cos(x)(4\cos^2(x) - 3) \tag{3}$$

$$\sin(3x) = \sin(x)(3 - 4\sin^2(x)). \tag{4}$$

Question 3

Now compare it with the kink angle obtain from the maximum hoop stress (MHS) criterion remembering that near-tip stress fields can be approximate using stress intensity factors as

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left[\frac{3}{4}\cos(\frac{\alpha}{2}) + \frac{1}{4}\cos(\frac{3\alpha}{2})\right] + \frac{K_{II}}{\sqrt{2\pi r}}\left[-\frac{3}{4}\sin(\frac{\alpha}{2}) - \frac{3}{4}\sin(\frac{3\alpha}{2})\right], \quad (5)$$

$\sigma_{\theta\theta}$ being the hoop stress defined in polar coordinates.

Question 4

Finally express the third criterion for crack kinking. Write down the equations but solve them later using your favorite symbolic solver.

2 Plastic zone around a crack tip

Until now, we have mainly worked on asymptotic solutions, i.e. assuming infinite stresses at the crack tip. No material is however able to withstand the self-similar singular stress field theoretically caused by crack. At the very vicinity of tip, one should admit a region, the fracture process zone (FPZ) where linear elasticity breaks down. The near-tip square-root singular approximation can still be used if the size of this process zone stays small enough compared to other dimensions. In this exercise, we will estimate the radius and shape of the FPZ around a mode-I crack within a material following Von Mises plasticity.

Question 1

Recall the singular stress field σ_{xx}, σ_{yy}, σ_{zz}, τ_{xy}, τ_{xz} and τ_{yz} ahead of a Mode-I crack tip under plane stress conditions.

Question 2

Using Mohr's circle relationship, express the principal stresses σ_1, σ_2, and σ_3 resulting from the Mode-I singular stress field.

Question 3

Using Von Mises criterion, defined the limit of the plastic zone using the uniaxial yield strength σ_{YS}.

Question 4

Which effect are we neglecting in this estimation of the plastic zone ?

3 J-integral and fracture toughness

A narrow strip of material with elastic modulus E, poisson's ratio ν and height h is rigidly attached to parallel platens, as shown in Figure 2. If the upper platen is displaced by δ, determine the geometric stress intensity factor for an edge crack between the platen faces.

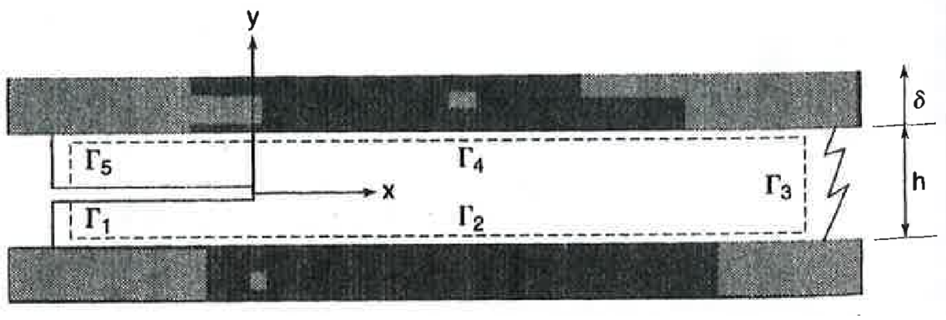

Figure 2: Edge crack in a strip of material.

Question 1

Decompose the J integral into subpart for each boundary of the contour Γ. Show that the value of J is only dependent on Γ_3. Show that:

$$J = \int_0^h W dy \tag{6}$$

Question 2

Using the geometry and Hooke's law, find the formula of ϵ_y and σ_y.
Hint: you might assume the problem is in plane stress conditions.

Question 3

Replace ϵ_y and σ_y in the J-integral and find the expression of J in terms of the displacement δ, the height h, the elastic modulus E and the poisson's ratio ν.

Question 4

Using the J-integral, find the expression of the stress intensity factor K.

Question 5

You want to design a joint with a material stress intensity factor K_C of 50 MPa.m$^{1/2}$. The material has a poisson's ratio of 0.3 and an elastic modulus of 200 GPa. Knowing that the maximum vertical elongation of the joint is 0.1 mm, what should be the height of the joint to resist crack propagation?

4 Singularity dominated zone

As we have seen in class, the stress field around a crack tip is singular. This means that the stress field diverges as we approach the crack tip. This singularity is a result of the assumption of linear elastic fracture mechanics (LEFM), which assumes that the material is elastic. In reality, materials are not perfectly elastic and will yield when the stress exceeds a certain limit. This leads to the formation of a plastic zone around the crack tip, where the material has yielded and the stress field is no longer singular. The size of this plastic zone is a function of the material properties and the applied stress. In this exercise, we will estimate the limit of LEFM and Irwin's approximation.

For this exercise, we will consider a mode-I crack in an infinite plate subjected to remote uniaxial stress σ_0. We assume that we are in plain strain conditions.

Question 1

Find a relation between the exact stresses solution and K_I when we are close to the crack tip. The full solution for the stresses on the crack plane for $\theta = 0$ is given by:

$$\sigma_{xx} = \frac{\sigma_0(a+r)}{\sqrt{2ar+r^2}}$$
$$\sigma_{yy} = \frac{\sigma_0(a+r)}{\sqrt{2ar+r^2}} - \sigma_0$$

Question 2

Compare the full solution for the stress field and the approximation you found in the previous question. When does the approximation of the stresses break down by more than 20%? Is it overestimated or underestimated by the approximation?

Question 3

Using the results of the previous question, estimate the size of the singularity-dominated zone r_s. Then, estimate the value of K_I where the plane strain plastic zone engulfs the singularity-dominated zone. What is the the maximal value of σ_0 regarding the yield strength σ_{YS} of the material to ensure that the approximation is valid?
Hint: you can use $\sigma_{xx} = \sqrt{3}\sigma_{YS}$.